Grades 6-8 Mathematics Item Specification Claim 3	
This claim refers to a recurring theme in the CCSSM content and practice standards: the ability to construct and present a clear, logical, convincing argument. For older students this may take the form of a rigorous deductive proof based on clearly stated axioms. For younger students this will involve more informal justifications. Assessment tasks that address this claim will typically present a claim or a proposed solution to a problem and will ask students to provide, for example, a justification, an explanation, or counter-example. (*Mathematics Content Specifications, p.63*) Communicating mathematical reasoning is not just a requirement of the Standards for Mathematical Practice—it is also a recurrent theme in the Standards for Mathematical Content. For example, many content standards call for students to explain, justify, or illustrate.	
Primary Claim 3: Communicating Reasoning: Students clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.	
Secondary Claim(s): Items/tasks written primarily to assess Claim 3 will necessarily involve Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 3 targets in the item form. If Claim 2 or Claim 4 targets are also directly related to the item/task, list those following the Claim 1 targets in order of prominence.	
Primary Content Domain: Each item/task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to and including the targeted grade within and across domains.	
Secondary Content Domain(s): While tasks developed to assess Claim 3 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate. The standards in the NS domain in grades 6-8 can be used to construct higher difficulty items for the adaptive pool. The integration of the RP, EE, F, and G domains with NS allows for higher content limits within the grade level than might be allowed when staying within the primary content domain.	
DOK Levels Target(s)	2, 3, 4
Allowable Response Types	**Response Types:** Multiple-Choice, single correct response (MC); Multiple Choice, multiple correct response (MS); Equation/Numeric (EQ); Drag and Drop, Hot Spot, and Graphing (GI); Matching Tables (MA); Fill-in Table (TI) No more than six choices in MS and MA items. Short Text - Performance tasks and Target B only **Scoring:** Scoring rules and answer choices will focus students' ability to use the appropriate reasoning. For some problems, multiple correct responses are possible. • MC will be scored as correct/incorrect (1 point) • If MS and MA items require two skills, scored as:

	o All correct choices (2 points); at least ½ but less than all correct choices (1 point) o Justification[1] for more than 1 point **must be** clear in the scoring rules o Where possible, include a "disqualifier" option that if selected would result in a score of 0 points, whether or not the student answered ½ correctly • EQ, GI, and TI items will be scored as: o Single requirement items will be scored as correct/incorrect (1 point) o Multiple requirement items: All components correct (2 points); at least ½ but less than all correct (1 point) o Justification for more than 1 point **must be** clear in the scoring rules
Allowable Stimulus Materials	Effort must be made to minimize the reading load in problem situations. Use tables, diagrams with labels, and other strategies to lessen reading load. Use simple subject-verb-object (SVO) sentences; use contexts that are familiar and relevant to all or most students at the targeted grade level. Target specific stimuli will be derived from the Claim 1 targets used in the problem situation.
Construct-Relevant Vocabulary	Refer to the Claim 1 specifications to determine construct-relevant vocabulary associated with specific content standards.
Allowable Tools	Any mathematical tools appropriate to the problem situation and the Claim 1 target(s). Some tools are identified in Standard for Mathematical Practice 5 and others can be found in the language of specific standards.
Target-Specific Attributes	CAT items should take from 2 to 5 minutes to solve; Claim 3 items that are part of a performance task may take 3 to 10 minutes to solve.
Accessibility Guidance	Item writers should consider the following Language and Visual Element/Design guidelines[2] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary

[1] For a CAT item to score multiple points, either distinct skills must be demonstrated that earn separate points or distinct levels of understanding of a complex skill must be tied directly to earning one or more points.

[2] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

	• Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[3]
Development Notes	• Items and task assessing Claim 3 may involve application of more than one standard. The focus is on communicating reasoning rather than demonstrating mathematical concepts or simple applications of mathematical procedures. • Targeted content standards for Claim 3 should belong to the major work of the grade (reference table of standards shown below). • Claim 1 *Specifications* that cover the following standards should be used to help inform an item writer's understanding of the difference between how these standards are measured in Claim 1 versus Claim 3. Development notes have been added to many of the Claim 1 specifications that call out specific topics that should be assessed under Claim 3. • Claim 3 items that require any degree of hand scoring must be written to primarily assess Target B. At least 80% of the items written to Claim 3 should primarily assess the standards and clusters listed in the table that follows.

Grade 6	Grade 7	Grade 8
6.RP.A	7.RP.A.2	8.EE.A.1
6.RP.A.3	7.NS.A	8.EE.B.5
6.NS.A	7.NS.A.1	8.EE.B.6
6.NS.A.1	7.NS.A.2	8.EE.C.7a
6.NS.C	7.EE.A.1	8.EE.C.7b
6.NS.C.5	7.EE.A.2	8.EE.C.8a
6.NS.C.6		8.F.A.1
6.NS.C.7		8.F.A.2
6.EE.A		8.F.A.3
6.EE.A.3		8.G.A.1
6.EE.A.4		8.G.A.2
6.EE.B		8.G.A.4
6.EE.B.6		8.G.A.5
6.EE.C.9		8.G.B.6
		8.G.B.8

[3] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Assessment Targets: Any given item/task should provide evidence for several of the following assessment targets; each of the following targets should not lead to a separate task. Multiple targets should be listed in order of prominence as related to the item/task.
Target A: Test propositions or conjectures with specific examples. (DOK 2) Tasks used to assess this target should ask for specific examples to support or refute a proposition or conjecture (e.g., An item might begin, "Provide 3 examples to show why/how…"). **Target B: Construct, autonomously[4], chains of reasoning that will justify or refute propositions or conjectures[5]. (DOK 3, 4)** Tasks used to assess this target should ask students to develop a chain of reasoning to justify or refute a conjecture. Tasks for Target B might include the types of examples called for in Target A as part of this reasoning, but should do so with a lesser degree of scaffolding than tasks that assess Target A alone. Some tasks for this target will ask students to formulate and justify a conjecture. **Target C: State logical assumptions being used. (DOK 2, 3)** Tasks used to assess this target should ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate. **Target D: Use the technique of breaking an argument into cases. (DOK 2, 3)** Tasks used to assess this target should ask students to determine under what conditions an argument is true, to determine under what conditions an argument is not true, or both. **Target E: Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in the argument—explain what it is. (DOK 2, 3, 4)** Tasks used to assess this target present students with one or more flawed arguments and ask students to choose which (if any) is correct, explain the flaws in reasoning, and/or correct flawed reasoning. **Target F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (DOK 2, 3)** In earlier grades, the desired student response might be in the form of concrete referents. In later grades, concrete referents

[4] By "autonomous" we mean that the student responds to a single prompt, without further guidance within the task.

[5] At the secondary level, these chains may take a successful student 10 minutes to construct and explain. Times will be somewhat shorter for younger students, but still giving them time to think and explain. For a minority of these tasks, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such "apprentice tasks," part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.

will often support generalizations as part of the justification rather than constituting the entire expected response.

Target G: At later grades, determine conditions under which an argument does and does not apply. (For example, area increases with perimeter for squares, but not for all plane figures.) (DOK 3, 4)

Tasks used to assess this target will ask students to determine whether a proposition or conjecture always applies, sometimes applies, or never applies and provide justification to support their conclusions. Targets A, B, C, and D will likely be included also in tasks that collect evidence for Target G.

Grade 6 standards that lend themselves to communicating reasoning	**The following standards can be effectively used in various combinations in Grade 6 Claim 3 items:** **Ratios and Proportional Relationships (RP)** **6.RP.A: Understand ratio concepts and use ratio reasoning to solve problems.** **6.RP.A.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. **The Number System (NS)** **6.NS.A: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.** **6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?* **6.NS.C: Apply and extend previous understandings of numbers to the system of rational numbers.** **6.NS.C.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. **6.NS.C.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. **6.NS.C.7** Understand ordering and absolute value of rational numbers. **Expressions and Equations (EE)** **6.EE.A: Apply and extend previous understandings of arithmetic to algebraic expressions.**

	6.EE.A.3 Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y. **6.EE.A.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for. **6.EE.B: Reason about and solve one-variable equations and inequalities.** **6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or, depending on the purpose at hand, any number in a specified set. **6.EE.C: Represent and analyze quantitative relationships between dependent and independent variables.** **6.EE.C.9** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.
Grade 7 standards that lend themselves to communicating reasoning	**The following standards can be effectively used in various combinations in Grade 7 Claim 3 items:** **Ratios and Proportional Relationships (RP)** **7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.** **7.RP.A.2** Recognize and represent proportional relationships between quantities. **The Number System (NS)** **7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.** **7.NS.A.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

	7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. **Expressions and Equations (EE)** **7.EE.A: Use properties of operations to generate equivalent expressions.** **7.EE.A.1** Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients. **7.EE.A.2** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that increase by 5% is the same as multiply by 1.05.
Grade 8 standards that lend themselves to communicating reasoning	**The following standards can be effectively used in various combinations in Grade 8 Claim 3 items:** **Expressions and Equations (EE)** **8.EE.A: Work with radicals and integer exponents** **8.EE.A.1** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. **8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.** **8.EE.B.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* **8.EE.B.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b. **8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.** **8.EE.C.7** Solve linear equations in one variable. **a.** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). **b.** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting

	like terms. **8.EE.C.8a** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. **Functions (F)** **8.F.A: Define, evaluate, and compare functions.** **8.F.A.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. **8.F.A.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions)…. **8.F.A.3** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear…. **Geometry (G)** **8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.** **8.G.A.1** Verify experimentally the properties of rotations, reflections, and translations: **8.G.A.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. **8.G.A.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. **8.G.A.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles…. **8.G.B: Understand and apply the Pythagorean Theorem.** **8.G.B.6** Explain a proof of the Pythagorean Theorem and its converse. **8.G.B.8** Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Range ALDs – Claim 3 Grades 6 - 8	**Level 1** Students should be able to base arguments on concrete referents such as objects, drawings, diagrams, and actions and identify obvious flawed arguments in familiar contexts.
	Level 2 Students should be able to find and identify the flaw in an argument by using examples or particular cases. Students should be able to break a familiar argument given in a highly scaffolded situation into cases to determine when the argument does or does not hold.
	Level 3 Students should be able to use stated assumptions, definitions, and previously established results and examples to test and support their reasoning or to identify, explain, and repair the flaw in an argument. Students should be able to break an argument into cases to determine when the argument does or does not hold.
	Level 4 Students should be able to use stated assumptions, definitions, and previously established results to support their reasoning or repair and explain the flaw in an argument. They should be able to construct a chain of logic to justify or refute a proposition or conjecture and to determine the conditions under which an argument does or does not apply.

Target 3A: Test propositions or conjectures with specific examples.

General Task Model Expectations for Target 3A

- Items for this target should focus on the core mathematical work that students are doing around ratios and proportional relationships, the rational number system, and equations and expressions in grades 6-7 and equations, functions, and geometry in grade 8.
- In response to a claim or conjecture, the student should:
  - Find a counterexample if the claim is false,
  - Find examples and non-examples if the claim is sometimes true, or
  - Provide supporting examples for a claim that is always true without concluding that the examples establish that truth, unless there are only a finite number of cases and all of them are established one-by-one. The main role for using specific examples in this case is for students to develop a hypothesis that the conjecture or claim is true, setting students up for work described in Claim 3B.
- False or partially true claims that students are asked to find counterexamples for should draw upon frequently held mathematical misconceptions whenever possible.
- Note: When asking students for a single example, take care to avoid mathematical language that suggests a single example proves a conjecture.
- Tasks have DOK Level 2.

Task Model 3A.1

- The student is presented with a proposition or conjecture and asked to give
  - a counterexample if the claim is false,
  - examples and non-examples if the claim is sometimes true, or
  - one or more supporting examples for a claim that is always true without concluding that the example(s) establish that truth.

Example Item 3A.1a (Grade 6)
Primary Target 3A (Content Domain NS), Secondary Target 1D (CCSS 6.NS.C), Tertiary Target 3G

Linh said, "The opposite of 5 is -5. The opposite of $\frac{2}{3}$ is $-\frac{2}{3}$. I think the opposite of a number is always negative." Linh's claim is **not** true. Give an example of a number whose opposite is **not** a negative number. Enter your answer in the response box.

Rubric: (1 point) The student enters a negative number or 0 in the response box.

Response Type: Equation/Numeric

Example Item 3A.1b (Grade 7)
Primary Target 3A (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 3G

When you divide 100 by a positive whole number, the result is always less than or equal to 100. This is not always true when you divide by a positive fraction.

Give an example of a fraction $\frac{a}{b}$ where $100 \div \frac{a}{b} < 100$
Enter your fraction in the first response box.

Give an example of a fraction $\frac{c}{d}$ where $100 \div \frac{c}{d} > 100$
Enter your fraction in the second response box.

Rubric: (1 point) The student enters appropriate fractions in the response boxes ($\frac{a}{b} > 1$ and $\frac{c}{d} < 1$)

Response Type: Equation/Numeric

Task Model 3A.2

- The student is presented with one or more propositions or conjectures and several examples and asked which examples support or refute one or more of the propositions.
- Items in this task model should cover all cases and not be unintentionally misleading about the truth status of a particular proposition or conjecture.

Example Item 3A.2a (Grade 6)

Primary Target 3A (Content Domain NS), Secondary Target 1D (CCSS 6.NS.C), Tertiary Target 3G

Gina said, "For every possible value of n, we know that $|-n| = n$."

Marco said, "Sometimes $|-n| = -n$."

Who is correct?

A. Gina
B. Marco

Select **all** the values for n shown below that support the correct claim.

B. $n = 12$
C. $n = 4.5$
D. $n = \frac{1}{2}$
E. $n = -4.5$
F. $n = -100$

Rubric: (1 point) The student selects the correct student (B, Marco) and all of the correct values that support Marco's claim (E and F).

Response Type: Multiple Choice, multiple correct response

Example Item 3A.2b (Grade 8)
Primary Target 3A (Content Domain NS), Secondary Target 1B (CCSS 7.NS.A), Tertiary Target 3G

Franco said that for any values *a*, *b*, and *c* the equation $a^2 + b^2 = c^2$ is always true. Mary disagrees.

Which of the following values for *a*, *b*, and *c* support Mary's claim? Select **all** that apply.

A. $a = 6$, $b = 8$, $c = 10$
B. $a = 2$, $b = 4$, $c = 6$
C. $a = b = c = 0$
D. $a = -2$, $b = 2$, $c = 0$

Rubric: (1 point) The student selects all of the correct values that support Mary's claim (B, D).

Response Type: Multiple choice, multiple correct response

Target 3B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.

General Task Model Expectations for Target 3B

- Items for this target should focus on the core mathematical work that students are doing around ratios and proportional relationships, the rational number system, and equations and expressions in grades 6-7 and equations, functions, and geometry in grade 8 with mathematical content from other domains playing a supporting role in setting up the reasoning contexts.
- Items for this target can probe a key mathematical structure such as that found in expressions and equations, ratios and proportional relationships, and the rational number system.
- Items for this target can require students to solve a multi-step, well-posed problem involving the application of mathematics to a real-world context. The difference between items for Claim 2A and Claim 3B is that the focus in 3B is on communicating the reasoning process in addition to getting the correct answer.
- Note that in grades 6-8, items provide less structure than items for earlier grades to focus on justifying or refuting a proposition or conjecture.
- Many machine-scorable items for these task models can be adapted to increase the autonomy of student's reasoning process but would require hand-scoring.
- Tasks have DOK Level 3, 4.

Task Model 3B.1

- The student is presented with a proposition or conjecture. The student is asked to identify or construct reasoning that justifies or refutes the proposition or conjecture.
- Items in this task model often address more generalized reasoning about a class of problems or reasoning that generalizes beyond the given problem context even when it is presented in a particular case.

Example Item 3B.1a (Grade 6)
Primary Target 3B (Content Domain NS), Secondary Target 1D (CCSS 6.NS.C, 4.G.A), Tertiary Target 3C

Lola said, "If n is a positive number, then the points $P = (n, n)$, $Q = (-n, n)$, $R = (-n, -n)$, and $S = (n, -n)$ are the vertices of a square in the coordinate plane."

Select **all** of the statements that support Lola's claim that the figure is a square.

A. The number n is a whole number.
B. The angles at P, Q, R and S, are all 90 degrees.
C. The distances between P and Q, Q and R, R and S, and S and P are n units.
D. The distances between P and Q, Q and R, R and S, and S and P are $2n$ units.

Rubric: (1 point) The student selects all of the statements that support Lola's claim (B and D).

Response Type: Multiple Choice, multiple correct response

Example Item 3B.1b (Grade 8)
Primary Target 3B (Content Domain NS), Secondary Target 1B (CCSS 7.NS.A), Tertiary Target 3C

The numbers *a*, *b*, and *c* are **not** zero and $a \bullet b = c$.

Part A
Click on the equation below that **must** also be true.

A. $-a \bullet b = c$
B. $a \bullet -b = c$
C. $-a \bullet -b = c$
D. $-a \bullet -b = -c$

Part B
Choose **four** statements that support your claim.

A. $-a = (-1) \bullet a$
B. $-b = (-1) \bullet b$
C. $-c = (-1) \bullet c$
D. $(-1) \bullet (-1) = 1$
E. $(-1) \bullet (1) = -1$
F. You can multiply numbers in any order.

Rubric: (2 point) The student selects the correct equation (C) and selects four statements that support the claim (A, B, D, and F).
(1 point) The student does one or the other.

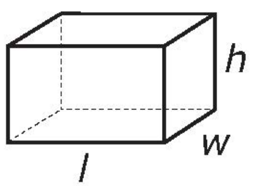
Response Type: Multiple choice, single correct response and multiple choice, multiple correct response

Task Model 3B.2

- The student is asked a mathematical question and is asked to identify or construct reasoning that justifies his or her answer.
- Items in this task model often address more generalized reasoning about a class of problems or reasoning that generalizes beyond the given problem context even when it is presented in a particular case.

Example Item 3B.2a (Grade 6)
Primary Target 3B (Content Domain G), Secondary Target 1H (CCSS 6.G.A), Tertiary Target 3A

A right rectangular prism has a height of 5 centimeters. Is it possible that the volume of the prism is 42 cubic centimeters?



(Not drawn to scale)

<u>If it is possible:</u>

Enter a possible length and width, in cm, of a prism with a height of 5 cm in two response boxes.

<u>If it is **not** possible:</u>

Enter a possible volume (in cubic centimeters) and the corresponding length and width (in centimeters) in the response boxes.

Rubric: (1 point) The student enters dimensions that are possible (e.g., any two numbers whose product is 8.4).

Response Type: Equation/Numeric (2 response boxes)

Commentary: This item addresses the misconception that the side-lengths of a right rectangular prism must be whole numbers or the related misconception that if the product of two numbers is a whole number then each factor must also be a whole number. Sixth grade is the year where students address the key related concepts most directly.

Example Item 3B.2b (Grade 7)
Primary Target 3B (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A)

A robot moves at a constant speed. It travels n miles in t minutes. The robot's pace is the number of minutes it takes to travel one mile.

Part A

A. What is the robot's speed in miles per minute?
B. What is the robot's pace in minutes per mile?

Part B

If the robot's speed is greater than 1, then the pace is

A. Greater than 1.
B. Equal to 1.
C. Less than 1.
D. Cannot be determined.

Explain your reasoning.

Rubric: (2 points) The student enters the correct speed (n/t) in the first response box and the correct pace (t/n) in the second response box and selects the correct statement about the pace (C) and enters a correct explanation (see Examples below).
(1 point) The student gets Part A right or Part B right, but not both.

Example 1
If the speed a/b is greater than 1, then the pace b/a must be less than one. The speed and the pace are reciprocals. If a number is greater than 1, then its reciprocal is less than one and vice-versa.

Example 2
The speed is greater than 1, so a/b>1. If we multiply both sides by b we get a>b. If we divide both sides by a, we get 1>b/a, which is the pace. So the pace is less than 1.

Response Type: Equation/numeric, multiple choice single correct response, short answer.

Note: Functionality for this item type does not currently exist, but the item could be implemented with a single text box.

Example Item 3B.2c (Grade 8)
Primary Target 3B (Content Domain EE), Secondary Target 1D (CCSS 8.EE.C), Tertiary Target 3F, Quaternary Target 3G

Part A
Is it possible for three linear equations in *x* and *y* to have a solution common to all three? [drop-down choices: yes, no]

Part B
[If "yes" is selected] Use the Arrow tool to draw the graphs of three equations that have a common solution. Add a point that represents the common solution.

[If "no" is selected] Explain why this is not possible in the response box.

Interaction: The student has to select yes or no before seeing Part B. If the student selects "yes" then he/she sees the graphing tools and is asked to graph the system. If he/she selects "no" there is a text box that asks for an explanation as to it is not possible. The student can change his/her mind.

Rubric: (1 point) The student selects "yes" and draws three lines that intersect in a single point and places a point at the intersection of the three lines (it is allowable for the lines to coincide, but they have to draw three graphs).

Response Type: Drop-Down Menu[6] and Graphing/Short-Text

Note: Functionality for this item type does not currently exist but it could be implemented by showing Parts A and B simultaneously. When possible, the point of having a student try to explain his or her incorrect reasoning is that in the process of trying to construct an argument, he or she may self-correct.

Task Model 3B.3

- Items for this target require students to solve a multi-step, well-posed problem involving the application of mathematics to a real-world context.
- The difference between Claim 2 task models and this task model is that the student needs to provide some evidence of his/her reasoning. The difference between Claim 4 task models and this task model is that the problem is completely well posed and no extraneous information is given.

[6] Drop-Down Menu response type is not yet available in the Smarter Balanced item authoring tool, but it is a scheduled enhancement by 2017.

Example Item 3B.3a (Grade 6)
Primary Target 3B (Content Domain RP), Secondary Target 1A (CCSS 6.RP.A), Tertiary Target 3C

Clark biked 4 miles in 20 minutes. How far can he go in 2 hours if he bikes at this rate?

Enter your answer in the first response box.

Show how you would solve this problem with a table or an equation (choose one option).

Option 1: Table

Enter values in the table so that it shows the number of miles, *m*, Clark can bike in 2 hours at this rate.

Miles (*m*)							
Minutes							
Hours							

Option 2: Equation

Enter an equation that can be solved to find the number of miles, *m*, Clark can bike in 2 hours at this rate in the second response box.

Rubric: (2 points) The student enters the correct number of miles (24) and fills in the table with at least two columns, one of which contains the correct answer, or enters an equation that can be solved to find the answer (see examples below of each).
(1 point) The student does one of these parts correctly.

Example for Option 1

Miles	4	8	12	16	20	24	
Minutes	20	40	60	80	100	120	
Hours	1/3	2/3	1	4/3	5/3	2	

Example for Option 2

$2 \cdot 3 \cdot 4 = m$ *or* $4/20 = m/120$ or equivalent equation.

Response Type: Equation/Numeric and Fill-in Table

Note: The functionality for this kind of combination of item types does not currently exist, but is a scheduled enhancement for 2017.

Example Item 3B.3b (Grade 7)
Primary Target 3B (Content Domain EE), Secondary Target 1D (7.EE.B), Tertiary Target 3C

In February, the price of a gallon of gasoline increased by 23% from the price in January. In March, the price decreased by 11% from the price in February. In March, gas cost \$2.63 per gallon.

How much did a gallon of gasoline cost in January, in dollars? Round your answer to the nearest cent. Enter your answer in the response box.

Which equation shown can be solved to find x, the cost of gas in January?

A. $(0.11)(0.23)x = 2.63$
B. $(1.11)(1.23)x = 2.63$
C. $(0.89)(1.23)x = 2.63$
D. $(1.11)(0.77)x = 2.63$

Rubric: (2 points) The student enters the correct cost of a gallon of gas (2.40) and selects the correct equation (C).
(1 point) The student does one of these parts correctly.

Response Type: Equation/numeric and multiple choice, single correct response

Note: Current functionality doesn't allow for mixing equation/numeric and multiple choice, so in the meantime the first part could be made multiple choice.

Example Item 3B.3c (Grade 8)
Primary Target 3B (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 4F

A car is traveling at a constant speed and drove 75 miles in 1.5 hours. One mile is approximately 1.6 kilometers. Approximately how fast is the car traveling in kilometers per hour? Explain or show clear steps for how you determined your answer.

Rubric: (2 points) The student includes the correct numeric value in the response (80) and provides a coherent, complete explanation or sequence of computations that shows where this comes from (see Examples).
(1 point) The student enters the correct numeric value but does not provide a coherent explanation OR the student provides an incorrect speed and includes an explanation that shows an understanding of how the answer could be found, but with some computational errors or a small misstep in reasoning.

<u>Example 1</u>

Going 75 miles in 1.5 hours is the same as going 50 miles per hour.
50 miles is 50*1.6=80 km.
A car driving 50 miles per hour is driving 80 kilometers per hour.

<u>Example 2</u>

75 miles in 1.5 hours is 75/1.5 = 50 mi/hr.
50 mi/hr * 1.6 km/mi =80 km/hr.
The car is traveling at 80 kilometers per hour.

Response Type: Short Text (handscored)

Target 3C: State logical assumptions being used.

General Task Model Expectations for Target 3C

- Items for this target should focus on the core mathematical work that students are doing around ratios and proportional relationships, the rational number system, and equations and expressions in grades 6-7 and equations, functions, and geometry in grade 8.
- For some items, the student must explicitly identify assumptions that
  - Make a problem well-posed, or
  - Make a particular solution method viable.
- When possible, items in this target should focus on assumptions that are commonly made implicitly and can cause confusion when left implicit.
- For some items, the student will be given a definition and be asked to reason from that definition.
- Tasks are DOK Level 2, 3.

Task Model 3C.1

- The student is asked to identify an unstated assumption that would make the problem well-posed or allow them to solve a problem using a given method.

Example Item 3C.1a (Grade 6)

Primary Target 3C (Content Domain RP), Secondary Target 1A (CCSS 6.RP.A), Tertiary Target 3G

Lyla flew her radio-controlled airplane 500 feet in 20 seconds. She claims that the speed of her airplane was 25 feet per second during the flight. What assumption must Lyla make for her claim to be true?

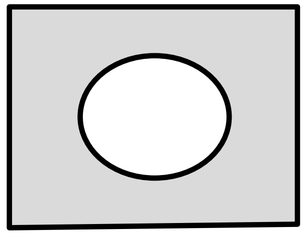
A. The airplane flew in a circle.
B. The airplane flew in a straight line.
C. The airplane flew at a constant speed.
D. The airplane flew faster at the end of the flight than at the beginning.

Rubric: (1 point) The student selects the correct statement (C).

Response Type: Multiple Choice, single correct response

Example Item 3C.1b (Grade 7)
Primary Target 3C (Content Domain G), Secondary Target 1F (CCSS 7.G.B), Tertiary Target 3G

Glenn saw the figure below and said,
"If I find the length (*l*), width (*w*), and radius (*r*), then the area (*A*) of the shaded region is $A = l \cdot w - \pi r^2$."



Which assumptions must Glenn be making in order for his equation to give the correct area of the shaded region? Select **all** that apply.

A. The quadrilateral is a rhombus.
B. The quadrilateral is a rectangle.
C. The curved figure in the center is a circle.
D. The curved figure in the center is a sphere.

Rubric: (1 point) The student selects the correct assumptions (B and C).

Response Type: Multiple Choice, single correct response

Task Model 3C.2

- The student will be given one or more definitions or assumptions and be asked to reason from that set of definitions and assumptions.

Example Item 3C.2a (Grade 7)
Primary Target 3C (Content Domain NS), Secondary Target 1B (CCSS 7.NS.A), Tertiary Target 3C

A **perfect square** is a number s that is the product of an integer, n, and itself, so that $s = n^2$.

Examples of perfect squares include 25 because it is equal to 5^2 and 81 because it is equal 9^2.

Can a perfect square be negative?

A. Yes; an example is –25.
B. No; a square of any integer is always positive.
C. Sometimes Yes, sometimes No; it depends on the value of n.
D. There is not enough information to tell.

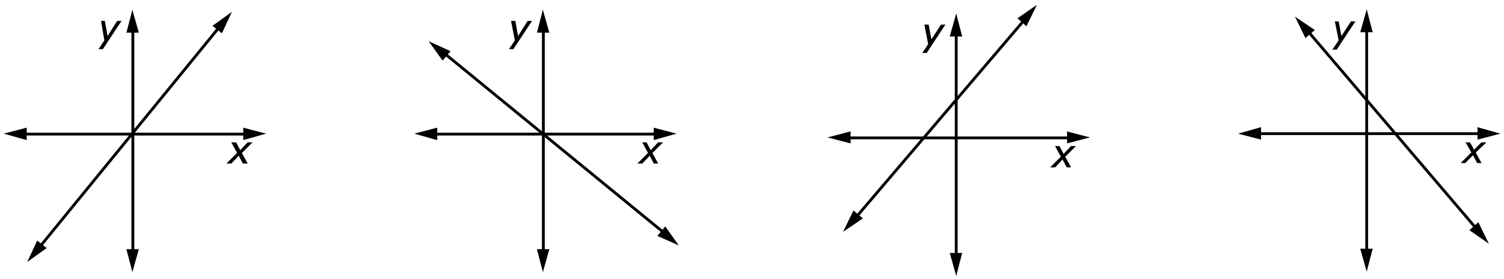
Rubric: (1 point) The student selects the correct statement (B).

Response Type: Multiple Choice, single correct response

Example Item 3C.2b (Grade 8)
Primary Target 3C (Content Domain EE), Secondary Target 1C (CCSS 8.EE.B), Tertiary Target 3F

A proportional relationship between x and y is one that can be represented by the equation $y = k \bullet x$, where k is a positive number.

Consider these four graphs.



Graph A **Graph B** **Graph C** **Graph D**

Based on this definition, identify whether or not each graph could represent a proportional relationship. Answer "Yes" if it does represent a proportional relationship and "No" if it does not.

	Yes	**No**
Graph A		
Graph B		
Graph C		
Graph D		

Rubric: (1 point) The student identifies the correct graphs (YNNN).

Response Type: Matching Table

Target 3D: Use the technique of breaking an argument into cases.

General Task Model Expectations for Target 3D

- Items for this target should focus on the core mathematical work that students are doing around ratios and proportional relationships, the rational number system, and equations and expressions in grades 6-7 and equations, functions, and geometry in grade 8.
- The student is given
  - a problem that has a finite number of possible solutions, some of which work and some of which don't, or
  - a proposition that is true in some cases but not others.
- Items for Claim 3 Target D should either present an exhaustive set of cases to consider or expect students to consider all possible cases in turn in order to distinguish it from items in other targets.
- Items have DOK Level 2, 3.

Task Model 3D.1

- The student is given a problem that has a finite number of possible solutions, some of which work and some of which don't.

Example Item 3D.1a (Grade 7)

Primary Target 3D (Content Domain RP), Second Target 1A (CCSS 7.RP.A), Tertiary Target 3G

Green paint can be made by mixing yellow paint with blue paint. Two mixtures make the same shade of green if the ratio of yellow to blue is the same. Assume n is a positive number.

Identify **one or more** of the mixtures below that will make the same shade of paint as a mixture of 10 liters of yellow paint and 15 liters of blue paint. Answer "Yes" if it will make the same shade of paint, answer "No" if it will not.

	Liters of Yellow Paint	**Liters of Blue Paint**	**Yes**	**No**
Mixture 1	$2n$	3		
Mixture 2	2	$3n$		
Mixture 3	$2n$	$3n$		

Rubric: (1 point) The student identifies the correct mixture (NNY).

Response Type: Matching Table

Note: A drag-and-drop version of this could allow students to determine the equivalent mixtures themselves.

Example Item 3D.1b (Grade 8)
Primary Target 3D (Content Domain G), Secondary Target 1G (CCSS 8.G.A), Tertiary Target 3G

Select **all** of the following situations that show that Figure *P* is congruent to Figure *Q*.

A. There is a translation that takes Figure *P* to Figure *Q*.
B. There is a rotation that takes Figure *P* to Figure *Q*.
C. There is a reflection that takes Figure *P* to Figure *Q*.
D. There is a dilation that takes Figure *P* to Figure *Q*.

Rubric: (1 point) The student selects the correct transformations (A, B, and C).

Response Type: Multiple choice, multiple selection response

Task Model 3D.2

- The student is given a proposition and asked to determine in which cases the proposition is true.

Example Item 3D.2a (Grade 7)
Primary Target 3D (Content Domain NS), Secondary Target 1B (CCSS 7.NS.A), Tertiary Target 3C

Given x and y are rational numbers, when is $|x + y| = |x| + |y|$ true?

A. This is never true.
B. This is always true.
C. This is true when x and y have opposite signs.
D. This is true when x and y have the same sign.

Rubric: (1 point) The student selects the correct statement (D).

Response Type: Multiple Choice, single correct response

Example Item 3D.2b (Grade 8)
Primary Target 3D (Content Domain EE), Secondary Target 1B (CCSS 8.EE.A), Tertiary Target 3C

Maggie claims that when you raise a whole number to a power, the result is always a greater number.
That is, $s^n > s$. For example:

$4^3 > 4$
$5^4 > 5$
$10^9 > 10$

Maggie's claim is **not** true for all values of *n* and *s*. For what values of *n* and *s* is Maggie's claim true? Complete the inequalities.

$s >$ [ ]

$n >$ [ ]

Rubric: (1 point) The student enters the correct values in the response boxes (1 and 1).

Response Type: Equation/Numeric (two response boxes, label the boxes with $s >$ and $n >$, respectively.)

Target 3E: Distinguish correct reasoning from flawed reasoning

General Task Model Expectations for Target 3E

- Items for this target should focus on the core mathematical work that students are doing around ratios and proportional relationships, the rational number system, and equations and expressions in grades 6-7 and equations, functions, and geometry in grade 8.
- The student is presented with valid or invalid reasoning and told it is flawed or asked to determine its validity. If the reasoning is flawed, the student identifies, explains, and/or corrects the error or flaw.
- The error should be more than just a computational error or an error in counting, and should reflect an actual error in reasoning.
- Analyzing faulty algorithms is acceptable so long as the algorithm is internally consistent and it isn't just a mechanical mistake executing a standard algorithm.
- Items have DOK Level 2, 3, 4.

Task Model 3E.1

- Some flawed reasoning or student work is presented and the student identifies and/or corrects the error or flaw.
- The student is presented with valid or invalid reasoning and asked to determine its validity. If the reasoning is flawed, the student will explain or correct the flaw.

Example Item 3E.1a (Grade 6)
Primary Target 3E (Content Domain EE), Secondary Target 1F (CCSS 6.EE.B), Tertiary Target 3C

Emma was solving the equation $t - 4 = 16$. She said, "I'm looking for a number *t* that is 4 less than 16. So $t = 12$."

Which statement best describes the flaw in Emma's reasoning?

A. Emma's answer is right but she should just subtract 4 from both sides of the equation.
B. Emma's answer is wrong but she thought about the equation correctly.
C. Emma is confused about which number the 4 is being subtracted from.
D. Emma should subtract the 16 from the 4 instead of 4 from the 16.

Rubric: (1 point) The student selects the correct analysis of the flaw in reasoning (C).

Response Type: Multiple choice, single correct response

Example Item 3E.1b (Grade 7)
Primary Target 3E (Content Domain RP), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 3C

Dena is trying to solve this problem:

> A store has a sale where every item has a sale price that is 20% less than the regular price. Write an expression that represents the sale price of an item if the regular price is *p* dollars.

Dena said, "To find 20% of a number, I should multiply by 0.20. So the sale price of an item will be $0.20p$."

Which statement best describes Dena's reasoning?

A. Dena is correct.
B. Dena needs to subtract 0.20p from the regular price, *p*.
C. Dena should calculate the sale price as $20p$ and then divide by 100.
D. Dena is trying to solve an impossible problem because it doesn't say what the regular price is.

Rubric: (1 point) The student selects the statement that represents correct reasoning (B).

Response Type: Multiple choice, single correct response

Task Model 3E.2

- Two or more approaches or chains of reasoning are given and the student is asked to identify the correct method and justification OR identify the incorrect method/reasoning and the justification.

Example Item 3E.2a (Grade 7)

Primary Target 3E (Content Domain NS), Secondary Target 1B (CCSS 6.NS.A), Tertiary Target 3C

Clyde and Lily were solving the equation $\frac{8}{9} \div \frac{1}{2} = x$.

Clyde said, "I can think of this division problem as a multiplication problem." Then he wrote:		Lily said, "You need to invert and multiply." Then she wrote:	
Step 1.	$\frac{8}{9} \div \frac{1}{2} = x$	Step 1.	$\frac{8}{9} \div \frac{1}{2} = x$
Step 2.	$\frac{1}{2}x = \frac{8}{9}$	Step 2.	$\frac{8}{9} = 2 \bullet x$
Step 3.	$2\left(\frac{1}{2}x\right) = 2\left(\frac{8}{9}\right)$	Step 3.	$\frac{1}{2}(2x) = \left(\frac{1}{2}\right) \bullet \left(\frac{8}{9}\right)$
Step 4.	$x = \frac{16}{9}$	Step 4.	$x = \frac{8}{18}$

Who solved the problem correctly?

A. Only Clyde solved the equation correctly.
B. Only Lily solved the equation correctly.
C. They both solved the equation correctly.
D. Neither one solved the equation correctly.

Rubric: (1 point) The student selects the correct characterization of these two approaches (A).

Response Type: Multiple choice, single correct response

Example Item 3E.2b (Grade 8)
Primary Target 3E (Content Domain EE), Secondary Target 1D (CCSS 8.EE.C), Tertiary Target 3C, Quaternary Target 3F

The students in Mr. Martin's class are learning about linear equations. Kenny made a claim and two supporting claims about the possible number of solutions to a system of linear equations. Rhonda made a different claim with two supporting claims.

Indicate whether each claim is valid or not valid.

Kenny's Claims	Valid	Not Valid
Claim 1. A system of two linear equations can only have zero solutions or one solution.		
Claim 1a. If the corresponding lines are distinct and parallel, then there are no solutions.		
Claim 1b. If the corresponding lines are distinct and intersect, then there is one solution.		

Rhonda's Claims	Valid	Not Valid
Claim 2. A system of two linear equations can have more than one solution.		
Claim 2a. If the corresponding lines intersect in exactly two places, then there will be exactly two solutions.		
Claim 2b. If the corresponding lines completely coincide, then there are an infinite number of solutions.		

Rubric: (1 point) The student selects the correct claims (NVV, VNV).

Response Type: Matching Table

Target 3F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions

Task Model 3F.1

- The student uses concrete referents to help justify or refute an argument.
- In grade 6, items in this task model should focus on the use of number lines. In grade 7, they should focus on number lines and graphs of proportional relationships. In grade 8, they should focus on graphs of linear equations and systems of linear equations and geometric contexts related to transformations of the plane or the Pythagorean Theorem.
- Items have DOK Level 2, 3.

Example Item 3F.1a (Grade 7)
Primary Target 3F (Content Domain NS), Secondary Target 1D (CCSS 6.NS.C), Tertiary Target 3D

P and T are numbers and $P + T = 0$.

Select **all** of the statements about P and Q that could be true.

A. $P = 0$ and $T = 0$

B. $P = 0$ or $T = 0$, but not both.

C. P can be any positive number and T can be any negative number.

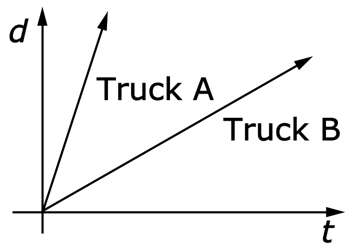
D. P and T are on opposite sides of zero and equally distant from zero on the number line.

Rubric: (1 point) The student selects the correct statements (A, D).

Response Type: Multiple Choice, multiple correct response

Example Item 3F.1b (Grade 7)
Primary Target 3F (Content Domain NS), Secondary Target 1A (CCSS 7.RP.A), Tertiary Target 3D

Two trucks are traveling on a highway at a constant speed. The graphs of their distances, *d*, over time, *t*, are shown.



Which truck is traveling faster, and how do you know?

Truck [drop-down menu choices: A, B] is traveling faster because the graph is [drop-down menu choices: steeper, less steep, longer, shorter].

Rubric: (1 point) The student chooses the correct truck (A) and the correct reason (steeper).

Response Type: Drop-down menu

Note: Functionality for this item type does not currently exist, but could be implemented with two-part multiple choice.

Example Item 3F.1c (Grade 8)
Primary Target 3F (Content Domain G), Secondary Target 1H (CCSS 8.G.B), Tertiary Target 3B

The Pythagorean Theorem states that if a right triangle has legs of length *a* and *b* and hypotenuse of length *c*, then $a^2 + b^2 = c^2$.

Figures 1 and 2 represent the key ideas in a proof of the Pythagorean Theorem.

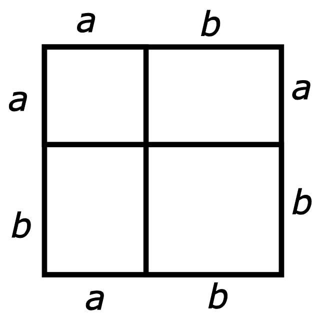


Figure 1

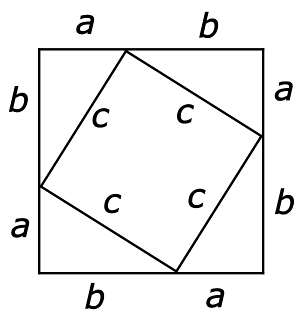


Figure 2

Create an outline a proof for the Pythagorean Theorem based on Figures 1 and 2, by dragging the seven statements shown into a logical sequence.

A right triangle has legs of length *a* and *b* and hypotenuse of length *c*.

1.

2.

3.

4.

5.

6.

7.

Thus, $a^2 + b^2 = c^2$

Subdivide the large square in Figure 1 into a square with side-length *a*, a square with side-length *b*, and two rectangles with side-lengths *a* and *b*.
Subdivide the large square in Figure 2 into four right triangles with legs *a* and *b* and a square with side-length *c*.
The total area of the large square in Figure 1 is $a^2 + b^2 + ab + ab$.
The total area of the large square in Figure 2 is $c^2 + 4(\frac{1}{2}ab)$.
Start with two large squares with sides of length $a + b$.
$a^2 + b^2 + ab + ab = c^2 + 4(\frac{1}{2}ab)$
The two large squares have the same area because they are congruent.

Rubric: (2 points) The student drags the steps of the proof into a logical order. Note that 1 must be first and 7 must be last and 2 must precede 5 and 3 must precede 6, but any other permutations are allowed as long as they are consistent with these constraints).
(1 point) The student gets the steps in an order consistent with the constraints described above, but has at most one step out of order.

Exemplar (more solutions are possible as noted above)

1. Start with two large squares with sides of length $a + b$.
2. Subdivide the large square in Figure 1 into a square with side-length a, a square with side length b, and two rectangles with side-lengths a and b.
3. Subdivide the large square in Figure 2 into four right triangles with legs a and b and a square with side-length c.
4. The two large squares have the same area because they are congruent.
5. The total area of the large square in Figure 1 is $a^2 + b^2 + ab + ab$.
6. The total area of the large square in Figure 2 is $c^2 + 4(\frac{1}{2}ab)$.
7. $a^2 + b^2 + ab + ab = c^2 + 4(\frac{1}{2}ab)$.

Response Type: Drag and Drop

Target 3G: Determine conditions under which an argument does and does not apply

Target 3G is a closely related extension of the expectations in Targets 3A, 3B, 3C, and 3D, and as with those targets, is often a tertiary alignment for items in those targets. Students often test propositions and conjectures with specific examples (as described in Target 3A) for the purpose of formulating conjectures about the conditions under which an argument does and does not apply. Students then must explicitly describe those conditions (as in Target 3C). Expectations for Target 3D include determining conditions under which an argument is true given cases—the next step is articulating those cases autonomously (Target 3B).